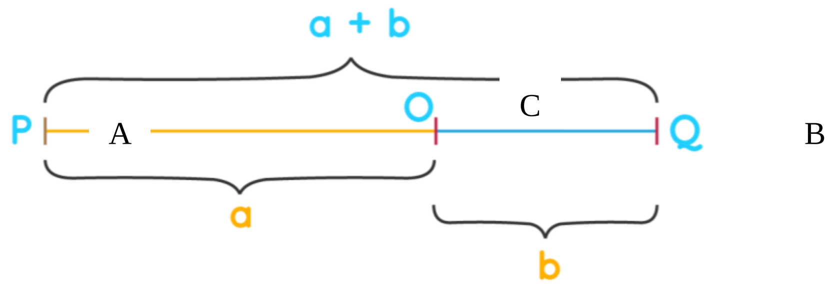
THE GOLDEN RATIO

Why? The perfect proportion (or golden section) of the body within a circle can be represented by a segment divided into two parts *a* and *b*, where *a+b* represents the total height of the person and *a* the distance from the ground to the navel, such that the ratio of *a+b* to *a* is equal to the ratio of *a* to *b*. From which we get that the golden section is: $1{,}618033988$.
Concretely, to have perfect proportions, a tall person $1.62m$ will have to have the navel about $1m$ from the ground.



$$\frac{a}{b} = \frac{a+b}{a} = 1.618.... = \phi$$

Educational Links: ***How to Calculate*** φ?

The golden section of segment AB is segment AC, with C between A and B, the proportional mean between the entire segment AB and the remaining part CB, i.e.

$$AB{:}AC = AC{:}CB$$
$$AB{:}AC = AC{:}(AB - AC)$$

We indicate:

$$AB = x$$
$$AC = a$$

$$x : a = a : (x - a)$$
$$a^2 = x \cdot (x - a)$$
$$x^2 - ax - a^2 = 0$$

This is a second-degree equation in the unknown x, which admits two solutions only one acceptable because it is positive having the value:

$$x_1 = a\frac{1+\sqrt{5}}{2}$$

At this point we define the *golden* ratio as the ratio $\varphi = \frac{x}{a}$ ie:

$$\varphi = \frac{1+\sqrt{5}}{2}$$

What is so special about it φ?

- The square of $\varphi$is equal to φincreased by 1: $\varphi^2 = \varphi + 1$
- The reciprocal of $\varphi$is equal to φdecreased by 1:
  $\frac{1}{\varphi} = \varphi - 1$
- The properties we have just outlined indicate, among other things, that the square and the reciprocal of φhave the same decimal part as φ:

$$\varphi = 1{,}6180339887 \ldots$$

$$\varphi^2 = 2{,}6180339887 \ldots$$

$$\frac{1}{\varphi} = 0{,}6180339887 \ldots$$

In addition, φ is the only number for which this situation (if we exclude natural numbers, of course).

In geometry, the golden angle is the angle subtended by the smallest arc of the circumference (the arc in red in the adjacent figure) that is obtained by dividing the circumference itself into two arcs that stand between them in the same ratio as in the golden section.
The value of this angle is $137^{\circ}\,30'$.